A proposal on the search for the hybrid with $I^G(J^{PC}) = 1^-(1^{-+})$ in the process $J/\psi \to \rho\omega\pi\pi$ at upgraded BEPC/BES *

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Abstract

The moment expressions for the boson resonances X with spin-parity $J_X^{P_X C} = 0^{++}$, 1^{-+}, 1^{++}, and 2^{++} possibly produced in the process $J/\psi \to \rho X$, $X \to b_1(1235)\pi$, $b_1 \to \omega\pi$ are given in terms of the generalized moment analysis method. The resonance with $J_X^{P_X C} = 1^{-+}$ can be distinguished from other resonances by means of these moments except for some rather special cases. The suggestion that the search for the hybrid with $I^G(J^{PC}) = 1^-(1^{-+})$ can be performed in the decay channel $J/\psi \to \rho\omega\pi\pi$ at upgraded BEPC/BES is presented.

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I. Introduction

Apart from the ordinary $q\bar{q}$ mesons, the new hadronic states such as glueballs (gg/ggg), hybrids ($q\bar{q}g$) and four-quark states ($qq\bar{q}\bar{q}$) also exist according to the predictions of QCD. Discovery and confirmation of any one of these new hadronic states would be the strong support to the QCD theory. Therefore, the search for and identifying these new hadronic states is a very excited and attractive research subject both theoretically and experimentally.

These new hadronic states can have the same quantum number J^{PC} as the ordinary $q\bar{q}$ mesons, what's more, they can also have the exotic quantum number J^{PC} which are not allowed in the quark model such as 1^{-+}, and thus they can not mix with the ordinary mesons. Experimentally, GAMS collaboration [1], E179 collaboration at KEK [2], VES group [3], E852 collaboration at BNL [4] and Crystal Barrel [5] all claimed that the evidence for the exotic state with $J^{PC}=1^{-+}$ was observed. The observed $\rho\pi$, $\eta\pi$ and $\eta'\pi$ couplings of this state qualitatively support the hypothesis that it is a hybrid meson, although other interpretations cannot be eliminated [6].

In terms of the predictions of the lattice QCD, the lowest lying glueball with $J^{PC}=1^{-+}$ has a higher mass than J/ψ [7]. Bag model calculations [8] predict that the lowest lying $qq\bar{q}\bar{q}$ states do not carry exotic quantum numbers and form nonets carrying the same quantum numbers as $q\bar{q}$ nonets, and that most $qq\bar{q}\bar{q}$ states can fall apart into two mesons and thus have a decay width in the order of their mass, which leads to that most $qq\bar{q}\bar{q}$ states are expected to be essentially unconfined and will not be observed as resonance peaks with reasonably narrow widths. Therefore, the search for the gluballs and four-quark states with $J^{PC}=1^{-+}$ at BEPC/BES could be disappointing. However, lattice QCD predicts that the mass of the hybrid with $J^{PC}=1^{-+}$ is $1.2\sim2.5$ GeV [8]. In addition, the naive estimate of pQCD predicts that the J/ψ hadronic decay processes are favorable to the production of hybrids. So, if the hybrids exist, the search for hybrid with $J^{PC}=1^{-+}$ at BEPC/BES should be fairly hopeful.

H. Yu and Q.X. Shen [9] have already discussed the possibility of the search for the hybrid with $J^{PC}=1^{-+}$ in the process $J/\psi\rightarrow\rho X,\ X\rightarrow\eta\pi\ (\eta'\pi,\ \rho\pi)$. For the decay modes of the

hybrid with $I^G(J^{PC}) = 1^-(1^{-+})$, according to the symmetrization selection rule [10, 11], the $\eta\pi$, $\eta'\pi$ modes are strongly suppressed (A possible mechanism to explain why the 1^{-+} state was observed in the above suppressed decay channels is planned for separated publication). The $\rho\pi$ mode is allowed, but this is a P-wave mode and thus the $\rho\pi$ mode should not be a dominant decay mode. The dominant decay mode should be the $b_1(1235)\pi$ [11]. Therefore, the probability of discovering the hybrid with $J^{PC} = 1^{-+}$ in the process $J/\psi \to \rho X$, $X \to b_1\pi$, in principle, should be higher than that in the process $J/\psi \to \rho X$, $X \to \eta\pi$ $(\eta'\pi,\ \rho\pi)$. Also, since the dominant decay mode of b_1 is $\omega\pi$, compared to the study on the two-step two-body decay process of J/ψ in Ref. [9, 12], the study on the three-step two-body decay process $J/\psi \to \rho X$, $X \to b_1\pi$, $b_1 \to \omega\pi$ perhaps could present more information to the experimentists. In this work, we shall consider the process $J/\psi \to \rho X$, $X \to b_1\pi$, $b_1 \to \omega\pi$.

This work is organized as follows. In Sect. II, we give the moment expressions for the resonances X with the above spin-parity in the process $J/\psi \to \rho X$, $X \to b_1\pi$, $b_1 \to \omega\pi$ in terms of the generalized moment analysis method [9, 12, 13]. , and in Sect. III, we discuss how to identify the resonances X with different spin-parity. Our conclusion is reached in Sect. IV.

II. Moment analysis

We consider the process

$$e^+ + e^- \to J/\psi \to \rho + X, \quad X \to b_1 + \pi, \quad b_1 \to \omega + \pi. \tag{1}$$

The S matrix element of the process (1) can be written as

$$\langle \rho_{\lambda_\rho} \omega_{\lambda_\omega} \pi\pi | S - 1 | e_r^+ e_{r'}^- \rangle \propto \langle \psi_{\lambda_J} | T | e_r^+ e_{r'}^- \rangle \langle \rho_{\lambda_\rho} X_{\lambda_X} | T_1 | \psi_{\lambda_J} \rangle \langle b_{1\lambda_{b_1}} \pi | T_2 | X_{\lambda_X} \rangle \langle \omega_{\lambda_\omega} \pi | T_3 | b_{1\lambda_{b_1}} \rangle, \tag{2}$$

where

$$\langle \psi_{\lambda_J} | T | e_r^+ e_{r'}^- \rangle \propto e_\mu^{\lambda_J *}(\vec{p}_J) \bar{v}_r(\vec{p}_+) \gamma^\mu u_{r'}(\vec{p}_-); \tag{3}$$

$$\langle \rho_{\lambda_\rho} X_{\lambda_X} | T_1 | \psi_{\lambda_J} \rangle \propto A^{J_X}_{\lambda_\rho, \lambda_X} D^{1*}_{\lambda_J, \lambda_\rho - \lambda_X}(0, \theta_\rho, 0); \tag{4}$$

$$\langle b_{1\lambda_{b_1}} \pi | T_2 | X_{\lambda_X} \rangle \propto B^{J_X}_{\lambda_{b_1}} D^{J_X *}_{\lambda_X, \lambda_{b_1}}(\phi_1, \theta_1, -\phi_1); \tag{5}$$

$$\langle \omega_{\lambda_\omega} \pi | T_3 | b_{1\lambda_{b_1}} \rangle \propto C_{\lambda_\omega} D^{1*}_{\lambda_{b_1},\lambda_\omega}(\phi_2, \theta_2, -\phi_2); \tag{6}$$

And λ_J, λ_ρ, λ_X, λ_{b_1} and λ_ω are the helicities of J/ψ, ρ, X, b_1 and ω, respectively; r and r' are the polarization indexes of the positron and electron, respectively; $\vec{p}_J$, $\vec{p}_+$, $\vec{p}_-$ are the momenta of J/ψ, positron and electron in the c.m. system of e^+e^-, respectively; $A^{J_X}_{\lambda_\rho,\lambda_X}$, $B^{J_X}_{\lambda_{b_1}}$ and C_{λ_ω} are the helicity amplitudes of the processed $J/\psi \to \rho X$, $X \to b_1\pi$ and $b_1 \to \omega\pi$, respectively; θ_ρ is the polar angle in the c.m. system of e^+e^- in which z axis is chosen to be along the direction of the incident positron and the vector meson ρ lies in $x - z$ plane; (θ_1, ϕ_1) describes the direction of the momentum of b_1 in the rest frame of X where the z_1 axis is chosen to be along the direction of the momentum of X in the c.m. system of e^+e^-; Similarly, (θ_2, ϕ_2) described the direction of the momentum of the vector mesons ω in the rest frame of b_1 where the z_2 axis is along the momentum of b_1 in the rest frame of X; The function $D^J_{m,n}$ is the $(2J+1)$-dimensional representation of the rotation group. Owing to the parity conservation for the process (1), these helicity amplitudes satisfy the following symmetry relations [14]:

$$\begin{aligned}
&A^{J_X}_{-\lambda_\rho,-\lambda_X} = P_X(-1)^{J_X} A^{J_X}_{\lambda_\rho,\lambda_X},\\
&B^{J_X}_{-\lambda_{b_1}} = P_X(-1)^{J_X} B^{J_X}_{\lambda_{b_1}},\\
&C_{-\lambda_\omega} = C_{\lambda_\omega},
\end{aligned} \tag{7}$$

where P_X is the parity of X.

The angular distribution for the process (1) is

$$\begin{aligned}
&W(\theta_\rho, \theta_1, \phi_1, \theta_2, \phi_2) \propto\\
&\sum_{\lambda_J,\lambda'_J} \sum_{\lambda_X,\lambda'_X} \sum_{\lambda_{b_1},\lambda'_{b_1}} \sum_{\lambda_\rho,\lambda_\omega} I_{\lambda_J,\lambda'_J} A^{J_X}_{\lambda_\rho,\lambda_X} A^{J_X*}_{\lambda_\rho,\lambda'_X} B^{J_X}_{\lambda_{b_1}} B^{J_X*}_{\lambda'_{b_1}} C_{\lambda_\omega} C^*_{\lambda_\omega}\\
&\times D^{1*}_{\lambda_J,\lambda_\rho-\lambda_X}(0,\theta_\rho,0) D^1_{\lambda'_J,\lambda_\rho-\lambda'_X}(0,\theta_\rho.0)\\
&\times D^{J_X*}_{\lambda_X,\lambda_{b_1}}(\phi_1,\theta_1,-\phi_1) D^{J_X}_{\lambda'_X,\lambda'_{b_1}}(\phi_1,\theta_1,-\phi_1)\\
&\times D^{1*}_{\lambda_{b_1},\lambda_\omega}(\phi_2,\theta_2,-\phi_2) D^1_{\lambda'_{b_1},\lambda_\omega}(\phi_2,\theta_2,-\phi_2),
\end{aligned} \tag{8}$$

where the density matrix elements I_{λ_J,λ'_J} is

$$I_{\lambda_J,\lambda'_J} \equiv \frac{1}{4} \sum_{r,r'} \langle \psi_{\lambda_J} | T | e^+_r e^-_{r'} \rangle \langle \psi_{\lambda'_J} | T | e^+_r e^-_{r'} \rangle^* \propto 2|\vec{p}_+|^2 \delta_{\lambda_J,\lambda'_J} \delta_{\lambda_J,\pm 1}. \tag{9}$$

The moments for the process (1) can be defined by

$$
\begin{aligned}
&M(j,L,M,\ell,m) = \\
&\int d\theta_\rho \sin\theta_\rho d\theta_1 \sin\theta_1 d\phi_1 d\theta_2 \sin\theta_2 d\phi_2 W(\theta_\rho,\theta_1,\phi_1,\theta_2,\phi_2) \\
&\times D^j_{0,-M}(0,\theta_\rho,0) D^L_{M,m}(\phi_1,\theta_1,-\phi_1) D^\ell_{m,0}(\phi_2,\theta_2,-\phi_2).
\end{aligned}
\tag{10}
$$

Eq. (10) can be reduced to

$$
\begin{aligned}
&M(j,L,M,\ell,m) \propto \\
&\sum_{\lambda_J=\pm1} \sum_{\lambda_X,\lambda'_X} \sum_{\lambda_{b_1},\lambda'_{b_1}} \sum_{\lambda_\rho,\lambda_\omega} A^{J_X}_{\lambda_\rho,\lambda_X} A^{J_X *}_{\lambda_\rho,\lambda'_X} B^{J_X}_{\lambda_{b_1}} B^{J_X *}_{\lambda'_{b_1}} C_{\lambda_\omega} C^*_{\lambda_\omega} \\
&\times \langle 1\lambda_J j0|1\lambda_J\rangle \langle 1(\lambda_\rho-\lambda'_X) j(-M)|1(\lambda_\rho-\lambda_X)\rangle \\
&\times \langle J_X\lambda'_X LM|J_X\lambda_X\rangle \langle J_X\lambda'_{b_1} Lm|J_X\lambda_{b_1}\rangle \\
&\times \langle 1\lambda'_{b_1}\ell m|1\lambda_{b_1}\rangle \langle 1\lambda_\omega \ell 0|1\lambda_\omega\rangle,
\end{aligned}
\tag{11}
$$

where $\langle j_1m_1j_2m_2|j_3m_3\rangle$ is Clebsch-Gordan coefficients.

In the process $X \to b_1\pi$, if we restrict $\ell_f \leq 1$, where ℓ_f is the relative orbital angular momentum between b_1 and π, the quantum number $I^G(J_X^{P_XC})$ of X allowed by the parity-isospin conservation law in the process (1) are $1^-(1^{-+})$, $1^-(0^{++})$, $1^-(1^{++})$, and $1^-(2^{++})$. For the resonances X with $J_X^{P_XC} = 0^{++},\ 1^{-+},\ 1^{++}$, and 2^{++}, the nonzero moment expressions derived from Eq. (11) are shown in Appendix A, B, C, and D.

There are four, twenty-one, sixteen, and thirty-one nonzero moment expressions for $J_X^{P_XC} = 0^{++}$, 1^{-+}, 1^{++}, and 2^{++}, respectively. In the following section, we shall discuss how to identify the X with the above $J_X^{P_XC}$.

III. Discussion

Since the helicity amplitudes $|C_0|^2$ and $|C_1|^2$ are independent of the spin-parity of the resonance X, we find that if $|C_0|^2 \neq |C_1|^2$ the moment expressions have the following characteristics: For $J_X^{P_XC} = 0^{++}$, the moments is always equal to zero in the case $L > 0$ or $M > 0$ or $m > 0$; For $J_X^{P_XC} = 1^{++}$, the nonzero moments with $L = 0,1,2,\ M = 0,1,2$ and $m = 0,2$ exist but the

moments are zero in the case $m = 1$; For $J_X^{P_X C} = 1^{-+}$, the nonzero moments with $L = 0, 1, 2$, $M = 0, 1, 2$ and $m = 0, 2$ exist, the nonzero moments with $m = 1$ also exist; For $J_X^{P_X C} = 2^{++}$, apart from the nonzero moments with $L = 0, 1, 2$, $M = 0, 1, 2$ and $m = 0, 1, 2$, the nonzero moments with $L = 3, 4$ exist. Therefore, from these characteristics, we can easily identify the resonances X with $J_X^{P_X C} = 0^{++}$, 1^{-+}, 1^{++} and 2^{++} experimentally.

However, if $|C_0|^2 = |C_1|^2$, some of the preceding characteristics disappear, which leads to that the situations in the case $|C_0|^2 = |C_1|^2$ are more complex than those in the case $|C_0|^2 \neq |C_1|^2$. We will turn to the special case $|C_0|^2 = |C_1|^2$ below.

(A) $|C_0|^2 = |C_1|^2$, $|B_0^1|^2 \neq |B_1^1|^2$ and $3|B_0^2|^2 \neq 4|B_1^2|^2$

In this case, only for $J_X^{P_X C} = 2^{++}$, there are four nonzero moments with $L = 4$, so the resonance with $J_X^{P_X C} = 2^{++}$ can be distinguished from other resonances. Then, for $J_X^{P_X C} = 0^{++}$, there are only two nonzero moments with $L = 0$, and for $J_X = 1$, there are four nonzero moments with $L = 2$, in addition to two nonzero moments with $L = 0$, hence the resonance with $J_X = 0$ can also be distinguished from that with $J_X = 1$. Finally, to distinguish the resonance with $J_X^{P_X C} = 1^{-+}$ from that with $J_X^{P_X} = 1^{++}$, we consider the following moment expression $H \equiv \frac{1}{8}M(00000) - \frac{5}{4}M(02000) - \frac{5}{4}M(20000) + \frac{25}{2}M(22000)$ and find that the H satisfies

$$H \propto \begin{cases} 0, & (J_X^{P_X C} = 1^{++}), \\ -\frac{27}{4}(|A_{00}^1|^2|B_0^1|^2 - 2|A_{11}^1|^2|B_0^1|^2 - 2|A_{00}^1|^2|B_1^1|^2)|C_1|^2, & (J_X^{P_X C} = 1^{-+}). \end{cases} \tag{12}$$

Using Eq. (12), we can still distinguish the resonance X with $J_X^{P_X C} = 1^{-+}$ from that with $J_X^{P_X C} = 1^{++}$.

(B) $|C_0|^2 = |C_1|^2$, $|B_0^1|^2 \neq |B_1^1|^2$ and $3|B_0^2|^2 = 4|B_1^2|^2$

In this case, compared to the case (A), the numbers of the nonzero moments for $J_X^{P_X C} = 0^{++}$, 1^{-+} and 1^{++} remain unchange, but for $J_X^{P_X C} = 2^{++}$, the moments with $L = 4$ disappear, There are still only two nonzero moments with $L = 0$ for $J_X^{P_X C} = 0^{++}$ and six nonzero moments with $L = 0,\ 2$ not only for $J_X = 1$ but also for $J_X = 2$. In this case, owing to Eq. (12) remains unchange and

$$H \propto -\frac{15}{2}|A_{00}^2|^2|B_1^2|^2|C_1|^2 < 0 \quad (J_X^{P_X C} = 2^{++}), \tag{13}$$

the crucial point is to distinguish the resonance with $J_X^{P_X C} = 1^{-+}$ from that with $J_X^{P_X C} = 2^{++}$. We also find $H_1 \equiv \frac{1}{4}M(02000) - \frac{5}{2}M(22000)$ satisfies

$$H_1 \propto \begin{cases} 3(|A_{00}^2|^2 + |A_{11}^2|^2)|B_1^2|^2|C_1|^2 > 0, & (J_X^{P_X C} = 2^{++}), \\ \frac{9}{5}|A_{11}^1|^2|B_1^1|^2|C_1|^2 > 0, & (J_X^{P_X C} = 1^{++}), \\ -\frac{9}{5}(|A_{00}^1|^2 - |A_{11}^1|^2)(|B_1^1|^2 - |B_0^1|^2)|C_1|^2, & (J_X^{P_X C} = 1^{-+}). \end{cases} \tag{14}$$

So, if it is determined experimentally that $H > 0$ or $H_1 \leq 0$, from Eq.(12)$\sim$(14), the $J_X^{P_X C}$ of X must be 1^{-+}. However, if $H < 0$ or $H_1 > 0$, we can not distinguish the resonance with $J_X^{P_X C} = 2^{++}$ from that with $J_X^{P_X C} = 1^{-+}$.

(C) $|C_0|^2 = |C_1|^2$ and $|B_0^1|^2 = |B_1^1|^2$

In this case, there are only two nonzero moments with $L = M = \ell = m = 0$ both for $J_X^{P_X C} = 0^{++}$ and $J_X^{P_X C} = 1^{-+}$, there are two nonzero moments with $L = 0$ and four nonzero moments with $L = 2$ for $J_X^{P_X C} = 1^{++}$, and there are two nonzero moments with $L = 0$ and at least four nonzero moments with $L = 2$ for $J_X^{P_X C} = 2^{++}$. Therefore, the resonances with $J_X^{P_X C} = 1^{++}$ and 2^{++} can be distinguished from the resonance with $J_X^{P_X C} = 0^{++}$ (or 1^{-+}). But it is almost impossible to distinguish the resonance with $J_X^{P_X C} = 1^{-+}$ from that with $J_X^{P_X C} = 0^{++}$ except in the radiative J/ψ decay process. Because for the radiative J/ψ decay process $e^+ + e^- \to J/\psi \to \gamma X$, $X \to b_1\pi$, $b_1 \to \omega\pi$, $A_{00}^0 = A_{01}^0 = A_{00}^1 = A_{01}^1$, we find

$$M(00000) - 10M(20000) \propto \begin{cases} 0, & (J_X^{P_X C} = 0^{++}), \\ 108|A_{11}^1|^2|B_1^1|^2|C_1|^2, & (J_X^{P_X C} = 1^{-+}). \end{cases} \tag{15}$$

Obviously, using Eq. (15) we can distinguish the 0^{++} state from the 1^{-+} state in the radiative J/ψ decay process.

From the above discussions, we get that if $|C_0|^2 \neq |C_1|^2$ we can easily identify the resonances X with $J_X^{P_X C}$= 0^{++}, 1^{-+}, 1^{++} and 2^{++}, but if $|C_0|^2 = |C_1|^2$ and $|B_0^1|^2 = |B_1^1|^2$ (or $|C_0|^2 = |C_1|^2$ and $3|B_0^2|^2 = 4|B_1^2|^2$), the identification of the resonances X with $J_X^{P_X C}$=1^{-+} and 0^{++} (or 2^{++}) is very difficult. However, we also want to note the following two points: 1) Since the ratio of the helicity amplitudes for the process $b_1(1235) \to \omega\pi$, $|C_0|$ and $|C_1|$, can be measured experimentally in other process such as $J/\psi \to b_1\pi$, $b_1 \to \omega\pi$. The measurement of the ratio

of $|C_0|$ and $|C_1|$ can be first performed in order to confirm whether $|C_0|^2$ is equal to $|C_1|^2$ or not; 2) Even though $|C_0|^2 = |C_1|^2$, one could expect that the probability of the simultaneous appearance of $|C_0|^2 = |C_1|^2$ and $|B_0^1|^2 = |B_1^1|^2$ (or $|C_0|^2 = |C_1|^2$ and $3|B_0^2|^2 = 4|B_1^2|^2$) would be fairly small.

It is worth pointing out that the above moment expressions and the discussions are also valid for the process $J/\psi \to \gamma X$, $X \to b_1\pi$, $b_1 \to \omega\pi$ provided $A_{00}^0 = A_{01}^0 = A_{00}^1 = A_{01}^1 = A_{00}^2 = A_{01}^2 = 0$.

IV. Conclusion

The twenty-one nonzero moment expressions for $J_X^{P_X C} = 1^{-+}$ show the possibility of the resonance X with $J_X^{P_X C} = 1^{-+}$ produced in the process (1) exists. At the same time, we can easily distinguish it from other resonances except for some rather special cases. Therefore, generally speaking, if the 50 million J/ψ events in the upgraded BEPC/BES are obtained, the search for the hybrid with $J^{PC} = 1^{-+}$ in the process $J/\psi \to \rho X$, $X \to b_1\pi$, $b_1(1235) \to \omega\pi$ is feasible.

Appendix A: The nonzero moments for $J_X^{P_X C} = 0^{++}$

$$
\begin{aligned}
M(00000) &\propto 2(|A_{00}^0|^2 + 2|A_{10}^0|^2)|B_0^0|^2(|C_0|^2 + 2|C_1|^2),\\
M(00020) &\propto \frac{4}{5}(|A_{00}^0|^2 + 2|A_{10}^0|^2)|B_0^0|^2(|C_0|^2 - |C_1|^2),\\
M(20000) &\propto -\frac{2}{5}(|A_{00}^0|^2 - |A_{10}^0|^2)|B_0^0|^2(|C_0|^2 + 2|C_1|^2),\\
M(20020) &\propto -\frac{4}{25}(|A_{00}^0|^2 - |A_{10}^0|^2)|B_0^0|^2(|C_0|^2 - |C_1|^2).
\end{aligned}
$$

Appendix B: The nonzero moments for $J_X^{P_X C} = 1^{-+}$

$$
\begin{aligned}
M(00000) &\propto 2(|A_{00}^1|^2 + 2|A_{01}^1|^2 + 2|A_{10}^1|^2 + 2|A_{11}^1|^2)(|B_0^1|^2 + 2|B_1^1|^2)(|C_0|^2 + 2|C_1|^2),\\
M(00020) &\propto \frac{4}{5}(|A_{00}^1|^2 + 2|A_{01}^1|^2 + 2|A_{10}^1|^2 + 2|A_{11}^1|^2)(|B_0^1|^2 - |B_1^1|^2)(|C_0|^2 - |C_1|^2),
\end{aligned}
$$

$$M(02000) \propto \frac{4}{5}(|A^1_{00}|^2 - |A^1_{01}|^2 + 2|A^1_{10}|^2 - |A^1_{11}|^2)(|B^1_0|^2 - |B^1_1|^2)(|C_0|^2 + 2|C_1|^2),$$
$$M(02020) \propto \frac{4}{25}(|A^1_{00}|^2 - |A^1_{01}|^2 + 2|A^1_{10}|^2 - |A^1_{11}|^2)(2|B^1_0|^2 + |B^1_1|^2)(|C_0|^2 - |C_1|^2),$$
$$M(02021) \propto \frac{12}{25}(|A^1_{00}|^2 - |A^1_{01}|^2 + 2|A^1_{10}|^2 - |A^1_{11}|^2)Re(B^1_1 B^{1*}_0)(|C_0|^2 - |C_1|^2),$$
$$M(02022) \propto \frac{12}{25}(|A^1_{00}|^2 - |A^1_{01}|^2 + 2|A^1_{10}|^2 - |A^1_{11}|^2)|B^1_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(20000) \propto -\frac{2}{5}(|A^1_{00}|^2 - |A^1_{01}|^2 - |A^1_{10}|^2 + 2|A^1_{11}|^2)(|B^1_0|^2 + 2|B^1_1|^2)(|C_0|^2 + 2|C_1|^2),$$
$$M(20020) \propto -\frac{4}{25}(|A^1_{00}|^2 - |A^1_{01}|^2 - |A^1_{10}|^2 + 2|A^1_{11}|^2)(|B^1_0|^2 - |B^1_1|^2)(|C_0|^2 - |C_1|^2),$$
$$M(21121) \propto \frac{6}{25}Im(A^1_{01}A^{1*}_{00} + A^1_{10}A^{1*}_{11})Im(B^1_1 B^{1*}_0)(|C_0|^2 - |C_1|^2),$$
$$M(22000) \propto -\frac{2}{25}(2|A^1_{00}|^2 + |A^1_{01}|^2 - 2|A^1_{10}|^2 - 2|A^1_{11}|^2)(|B^1_0|^2 - |B^1_1|^2)(|C_0|^2 + 2|C_1|^2),$$
$$M(22020) \propto -\frac{2}{125}(2|A^1_{00}|^2 + |A^1_{01}|^2 - 2|A^1_{10}|^2 - 2|A^1_{11}|^2)(2|B^1_0|^2 + |B^1_1|^2)(|C_0|^2 - |C_1|^2),$$
$$M(22021) \propto -\frac{6}{125}(2|A^1_{00}|^2 + |A^1_{01}|^2 - 2|A^1_{10}|^2 - 2|A^1_{11}|^2)Re(B^1_1 B^{1*}_0)(|C_0|^2 - |C_1|^2),$$
$$M(22022) \propto -\frac{6}{125}(2|A^1_{00}|^2 + |A^1_{01}|^2 - 2|A^1_{10}|^2 - 2|A^1_{11}|^2)|B^1_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(22100) \propto -\frac{6}{25}Re(A^1_{01}A^{1*}_{00} - A^1_{11}A^{1*}_{10})(|B^1_0|^2 - |B^1_1|^2)(|C_0|^2 + 2|C_1|^2),$$
$$M(22120) \propto -\frac{6}{125}Re(A^1_{01}A^{1*}_{00} - A^1_{11}A^{1*}_{10})(2|B^1_0|^2 + |B^1_1|^2)(|C_0|^2 - |C_1|^2),$$
$$M(22121) \propto -\frac{18}{125}Re(A^1_{01}A^{1*}_{00} - A^1_{11}A^{1*}_{10})Re(B^1_1 B^{1*}_0)(|C_0|^2 - |C_1|^2),$$
$$M(22122) \propto -\frac{18}{125}Re(A^1_{01}A^{1*}_{00} - A^1_{11}A^{1*}_{10})|B^1_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(22200) \propto -\frac{6}{25}|A^1_{01}|^2(|B^1_0|^2 - |B^1_1|^2)(|C_0|^2 + 2|C_1|^2),$$
$$M(22220) \propto -\frac{6}{125}|A^1_{01}|^2(2|B^1_0|^2 + |B^1_1|^2)(|C_0|^2 - |C_1|^2),$$
$$M(22221) \propto -\frac{18}{125}|A^1_{01}|^2 Re(B^1_1 B^{1*}_0)(|C_0|^2 - |C_1|^2),$$
$$M(22222) \propto -\frac{18}{125}|A^1_{01}|^2|B^1_1|^2(|C_0|^2 - |C_1|^2).$$

Appendix C: The nonzero moments for $J_X^{P_X C} = 1^{++}$

$$M(00000) \propto 8(|A^1_{01}|^2 + |A^1_{10}|^2 + |A^1_{11}|^2)|B^1_1|^2(|C_0|^2 + 2|C_1|^2),$$
$$M(00020) \propto -\frac{8}{5}(|A^1_{01}|^2 + |A^1_{10}|^2 + |A^1_{11}|^2)|B^1_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(02000) \propto \frac{4}{5}(|A^1_{01}|^2 - 2|A^1_{10}|^2 + |A^1_{11}|^2)|B^1_1|^2(|C_0|^2 + 2|C_1|^2),$$

$$M(02020) \propto -\frac{4}{25}(|A^1_{01}|^2 - 2|A^1_{10}|^2 + |A^1_{11}|^2)|B^1_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(02022) \propto \frac{12}{25}(|A^1_{01}|^2 - 2|A^1_{10}|^2 + |A^1_{11}|^2)|B^1_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(20000) \propto \frac{4}{5}(|A^1_{01}|^2 + |A^1_{10}|^2 - 2|A^1_{11}|^2)|B^1_1|^2(|C_0|^2 + 2|C_1|^2),$$
$$M(20020) \propto -\frac{4}{25}(|A^1_{01}|^2 + |A^1_{10}|^2 - 2|A^1_{11}|^2)|B^1_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(22000) \propto \frac{2}{25}(|A^1_{01}|^2 - 2|A^1_{10}|^2 - 2|A^1_{11}|^2)|B^1_1|^2(|C_0|^2 + 2|C_1|^2),$$
$$M(22020) \propto -\frac{2}{125}(|A^1_{01}|^2 - 2|A^1_{10}|^2 - 2|A^1_{11}|^2)|B^1_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(22022) \propto \frac{6}{125}(|A^1_{01}|^2 - 2|A^1_{10}|^2 - 2|A^1_{11}|^2)|B^1_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(22100) \propto -\frac{6}{25}Re(A^1_{11}A^{1*}_{10})|B^1_1|^2(|C_0|^2 + 2|C_1|^2),$$
$$M(22120) \propto \frac{6}{125}Re(A^1_{11}A^{1*}_{10})|B^1_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(22122) \propto -\frac{18}{125}Re(A^1_{11}A^{1*}_{10})|B^1_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(22200) \propto -\frac{6}{25}|A^1_{01}|^2|B^1_1|^2(|C_0|^2 + 2|C_1|^2),$$
$$M(22220) \propto \frac{6}{125}|A^1_{01}|^2|B^1_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(22222) \propto -\frac{18}{125}|A^1_{01}|^2|B^1_1|^2(|C_0|^2 - |C_1|^2).$$

Appendix D: The nonzero moments for $J_X^{P_X C} = 2^{++}$

$$M(00000) \propto 2(|A^2_{00}|^2 + 2|A^2_{01}|^2 + 2|A^2_{10}|^2 + 2|A^2_{11}|^2 + 2|A^2_{12}|^2)(|B^2_0|^2 + 2|B^2_1|^2)(|C_0|^2 + 2|C_1|^2),$$
$$M(00020) \propto \frac{4}{5}(|A^2_{00}|^2 + 2|A^2_{01}|^2 + 2|A^2_{10}|^2 + 2|A^2_{11}|^2 + 2|A^2_{12}|^2)(|B^2_0|^2 - |B^2_1|^2)(|C_0|^2 - |C_1|^2),$$
$$M(02000) \propto \frac{4}{7}(|A^2_{00}|^2 + |A^2_{01}|^2 + 2|A^2_{10}|^2 + |A^2_{11}|^2 - 2|A^2_{12}|^2)(|B^2_0|^2 + |B^2_1|^2)(|C_0|^2 + 2|C_1|^2),$$
$$M(02020) \propto \frac{4}{35}(|A^2_{00}|^2 + |A^2_{01}|^2 + 2|A^2_{10}|^2 + |A^2_{11}|^2 - 2|A^2_{12}|^2)(2|B^2_0|^2 - |B^2_1|^2)(|C_0|^2 - |C_1|^2),$$
$$M(02021) \propto \frac{4\sqrt{3}}{35}(|A^2_{00}|^2 + |A^2_{01}|^2 + 2|A^2_{10}|^2 + |A^2_{11}|^2 - 2|A^2_{12}|^2)Re(B^2_1B^{2*}_0)(|C_0|^2 - |C_1|^2),$$
$$M(02022) \propto \frac{12}{35}(|A^2_{00}|^2 + |A^2_{01}|^2 + 2|A^2_{10}|^2 + |A^2_{11}|^2 - 2|A^2_{12}|^2)|B^2_1|^2(|C_0|^2 - |C_1|^2),$$
$$M(04000) \propto \frac{4}{63}(3|A^2_{00}|^2 - 4|A^2_{01}|^2 + 6|A^2_{10}|^2 - 4|A^2_{11}|^2 + |A^2_{12}|^2)(3|B^2_0|^2 - 4|B^2_1|^2)(|C_0|^2 + 2|C_1|^2),$$
$$M(04020) \propto \frac{8}{315}(3|A^2_{00}|^2 - 4|A^2_{01}|^2 + 6|A^2_{10}|^2 - 4|A^2_{11}|^2 + |A^2_{12}|^2)(3|B^2_0|^2 + 2|B^2_1|^2)(|C_0|^2 - |C_1|^2),$$
$$M(04021) \propto \frac{4\sqrt{10}}{105}(3|A^2_{00}|^2 - 4|A^2_{10}|^2 + 6|A^2_{10}|^2 - 4|A^2_{11}|^2 + |A^2_{12}|^2)Re(B^2_1B^{2*}_0)(|C_0|^2 - |C_1|^2),$$

$$M(04022) \propto \frac{8\sqrt{15}}{315}(3|A^2_{00}|^2 - 4|A^2_{10}|^2 + 6|A^2_{10}|^2 - 4|A^2_{11}|^2 + |A^2_{12}|^2)|B^2_1|^2(|C_0|^2 - |C_1|^2),$$

$$M(20000) \propto -\frac{2}{5}(|A^2_{00}|^2 - |A^2_{01}|^2 - |A^2_{10}|^2 + 2|A^2_{11}|^2 - |A^2_{12}|^2)(|B^2_0|^2 + 2|B^2_1|^2)(|C_0|^2 + 2|C_1|^2),$$

$$M(20020) \propto -\frac{4}{25}(|A^2_{00}|^2 - |A^2_{01}|^2 - |A^2_{10}|^2 + 2|A^2_{11}|^2 - |A^2_{12}|^2)(|B^2_0|^2 - |B^2_1|^2)(|C_0|^2 - |C_1|^2),$$

$$M(21121) \propto \frac{2}{25}[3Im(A^2_{01}A^{2*}_{00} + A^2_{10}A^{2*}_{11}) + \sqrt{6}Im(A^2_{12}A^{2*}_{11})]Im(B^2_1B^{2*}_0)(|C_0|^2 - |C_1|^2),$$

$$M(22000) \propto -\frac{2}{35}(2|A^2_{00}|^2 - |A^2_{01}|^2 - 2|A^2_{10}|^2 + 2|A^2_{11}|^2 + 2|A^2_{12}|^2)(|B^2_0|^2 + |B^2_1|^2)(|C_0|^2 + 2|C_1|^2),$$

$$M(22020) \propto -\frac{2}{175}(2|A^2_{00}|^2 - |A^2_{01}|^2 - 2|A^2_{10}|^2 + 2|A^2_{11}|^2 + 2|A^2_{12}|^2)(2|B^2_0|^2 - |B^2_1|^2)(|C_0|^2 - |C_1|^2),$$

$$M(22021) \propto -\frac{2\sqrt{3}}{175}(2|A^2_{00}|^2 - |A^2_{01}|^2 - 2|A^2_{10}|^2 + 2|A^2_{11}|^2 + 2|A^2_{12}|^2)Re(B^2_1B^{2*}_0)(|C_0|^2 - |C_1|^2),$$

$$M(22022) \propto -\frac{6}{175}(2|A^2_{00}|^2 - |A^2_{01}|^2 - 2|A^2_{10}|^2 + 2|A^2_{11}|^2 + 2|A^2_{12}|^2)|B^2_1|^2(|C_0|^2 - |C_1|^2),$$

$$M(22100) \propto -\frac{\sqrt{2}}{35}[\sqrt{6}Re(A^2_{01}A^{2*}_{00} - A^2_{11}A^{2*}_{10}) + 6Re(A^2_{12}A^{2*}_{11})](|B^2_0|^2 + |B^2_1|^2)(|C_0|^2 + 2|C_1|^2),$$

$$M(22120) \propto -\frac{2}{175}[\sqrt{3}Re(A^2_{01}A^{2*}_{00} - A^2_{11}A^{2*}_{10}) + 3\sqrt{2}Re(A^2_{12}A^{2*}_{11})](2|B^2_0|^2 - |B^2_1|^2)(|C_0|^2 - |C_1|^2),$$

$$M(22121) \propto -\frac{6}{175}[Re(A^2_{01}A^{2*}_{00} - A^2_{11}A^{2*}_{10}) + \sqrt{6}Re(A^2_{12}A^{2*}_{11})]Re(B^2_1B^{2*}_0)(|C_0|^2 - |C_1|^2),$$

$$M(22122) \propto -\frac{3\sqrt{2}}{175}[\sqrt{6}Re(A^2_{01}A^{2*}_{00} - A^2_{11}A^{2*}_{10}) + 6Re(A^2_{12}A^{2*}_{11})]|B^2_1|^2(|C_0|^2 - |C_1|^2),$$

$$M(22200) \propto -\frac{2}{35}[3|A^2_{01}|^2 + 2\sqrt{6}Re(A^2_{12}A^{2*}_{10})](|B^2_0|^2 + |B^2_1|^2)(|C_0|^2 + 2|C_1|^2),$$

$$M(22220) \propto -\frac{2}{175}[3|A^2_{01}|^2 + 2\sqrt{6}Re(A^2_{12}A^{2*}_{10})](2|B^2_0|^2 - |B^2_1|^2)(|C_0|^2 - |C_1|^2),$$

$$M(22221) \propto -\frac{6}{175}[\sqrt{3}|A^2_{01}|^2 + 2\sqrt{2}Re(A^2_{12}A^{2*}_{10})]Re(B^2_1B^{2*}_0)](|C_0|^2 - |C_1|^2),$$

$$M(22222) \propto -\frac{6}{175}[3|A^2_{01}|^2 + 2\sqrt{6}Re(A^2_{12}A^{2*}_{10})]|B^2_1|^2(|C_0|^2 - |C_1|^2),$$

$$M(23121) \propto \frac{6}{175}[2Im(A^2_{01}A^{2*}_{00} + A^2_{10}A^{2*}_{11}) + \sqrt{6}Im(A^2_{11}A^{2*}_{12})]Im(B^2_1B^{2*}_0)(|C_0|^2 - |C_1|^2),$$

$$M(23221) \propto -\frac{12\sqrt{5}}{175}Im(A^2_{12}A^{2*}_{10})Im(B^2_1B^{2*}_0)(|C_0|^2 - |C_1|^2),$$

$$M(24000) \propto -\frac{2}{315}(6|A^2_{00}|^2 + 4|A^2_{01}|^2 - 6|A^2_{10}|^2 - 8|A^2_{11}|^2 - |A^2_{12}|^2)(3|B^2_0|^2 - 4|B^2_1|^2)(|C_0|^2 + 2|C_1|^2),$$

$$M(24020) \propto -\frac{4}{1575}(6|A^2_{00}|^2 + 4|A^2_{01}|^2 - 6|A^2_{10}|^2 - 8|A^2_{11}|^2 - |A^2_{12}|^2)(3|B^2_0|^2 + 2|B^2_1|^2)(|C_0|^2 - |C_1|^2),$$

$$M(24021) \propto -\frac{2\sqrt{10}}{525}(6|A^2_{00}|^2 + 4|A^2_{01}|^2 - 6|A^2_{10}|^2 - 8|A^2_{11}|^2 - |A^2_{12}|^2)Re(B^2_1B^{2*}_0)(|C_0|^2 - |C_1|^2),$$

$$M(24022) \propto -\frac{4\sqrt{15}}{1575}(6|A^2_{00}|^2 + 4|A^2_{01}|^2 - 6|A^2_{10}|^2 - 8|A^2_{11}|^2 - |A^2_{12}|^2)|B^2_1|^2(|C_0|^2 - |C_1|^2),$$

$$M(24100) \propto -\frac{\sqrt{10}}{315}[6Re(A^2_{11}A^{2*}_{10} - A^2_{01}A^{2*}_{00}) + \sqrt{6}Re(A^2_{12}A^{2*}_{11})](3|B^2_0|^2 - 4|B^2_1|^2)(|C_0|^2 + 2|C_1|^2),$$

$$M(24120) \propto -\frac{2\sqrt{10}}{1575}[6Re(A_{11}^2A_{10}^{2*} - A_{01}^2A_{00}^{2*}) + \sqrt{6}Re(A_{12}^2A_{11}^{2*})](3|B_0^2|^2 + 2|B_1^2|^2)(|C_0|^2 - |C_1|^2),$$
$$M(24121) \propto -\frac{2}{105}[6Re(A_{01}^2A_{00}^{2*} - A_{11}^2A_{10}^{2*}) - \sqrt{6}Re(A_{12}^2A_{11}^{2*})]Re(B_1^2B_0^{2*})(|C_0|^2 - |C_1|^2),$$
$$M(24122) \propto -\frac{4}{105}[\sqrt{6}Re(A_{01}^2A_{00}^{2*} - A_{11}^2A_{10}^{2*}) - Re(A_{12}^2A_{11}^{2*})]|B_1^2|^2(|C_0|^2 - |C_1|^2),$$
$$M(24200) \propto -\frac{2\sqrt{10}}{315}[\sqrt{6}|A_{01}^2|^2 - 3Re(A_{12}^2A_{10}^{2*})](3|B_0^2|^2 - 4|B_1^2|^2)(|C_0|^2 + 2|C_1|^2),$$
$$M(24220) \propto -\frac{4\sqrt{10}}{1575}[\sqrt{6}|A_{01}^2|^2 - 3Re(A_{12}^2A_{10}^{2*})](3|B_0^2|^2 + 2|B_1^2|^2)(|C_0|^2 - |C_1|^2),$$
$$M(24221) \propto \frac{4}{105}[-\sqrt{6}|A_{01}^2|^2 + 3Re(A_{12}^2A_{10}^{2*})]Re(B_1^2B_0^{2*})(|C_0|^2 - |C_1|^2),$$
$$M(24222) \propto -\frac{4}{105}[2|A_{01}^2|^2 - \sqrt{6}Re(A_{12}^2A_{10}^{2*})]|B_1^2|^2(|C_0|^2 - |C_1|^2).$$

References

[1] D. Alde *et al.*, Phys. Lett. **B205** (1988) 397.

[2] H. Aoyagi *et al.*, Phys. Lett. **B314**(1993) 246.

[3] G.M. Beladidze *et al.*, Phys. Lett. **B313** (1993) 276.

[4] D.R. Thompson *et al.*, Phys. Rev. Lett. **79** (1997) 1630.

[5] A. Abele *et al.*, Phys. Lett. **B423** (1998) 175.

[6] P.R. Page, Phys. Lett. **B415** (1997) 205.

[7] G. Bali *et al.*, Phys. Lett. **B309** (1993) 378.

[8] R. Jaffe and K. Johnson, Phys. Lett. **B60** (1976) 201.; T. Barnes, F.E. Close and F. de Viron, Nucl. Phys. **B224** (1983) 241; N. Isgur *et al.*, Phys. Rev. Lett. **53** (1985) 67; P. Lacock *et al.*, Phys. Lett. **B401** (1997) 308.

[9] Hong Yu and Qixing Shen, Chin. Phys. Lett. **16** (1999) 484; High Energy Phys. Nucl. Phys. **23** (1999) 954.

[10] F.Iddir *et al.*, Phys. Lett. **B205**(1988) 564; C.B.Dover *et al.*, Phys. Lett. **B277**(1992) 23; P.R.Page, Phys. Lett. **B401** (1997) 313; **B402** (1997) 183.

[11] P.R.Page *et al.*, Phys.Rev. **D59** (1999) 034016.

[12] Qixing Shen, Ming Chao and Hong Yu, Commun. Theor. Phys. **31** (1999) 429.

[13] Hong Yu, Commun. Theor. Phys. **12** (1989) 229; Qixing Shen and Hong Yu, Phys. Rev. **D48** (1993) 2129; S.U.Chung, Phys. Rev. **169** (1968) 1342.

[14] M. Jacob and G.C. Wick, Ann. Phys. (N.Y.) (1959) 404.